Curious energy losses at corners of metallic inclusions.

A.-S. Bonnet-Ben Dhia[1,*], C. Carvalho[1], C. Chambeyron[1], L. Chesnel[2], P. Ciarlet Jr.[1], A. Nicolet[3], F. Zolla[3]

[1]POEMS (CNRS - ENSTA ParisTech - INRIA), Palaiseau, France.
[2]CMAP (CNRS - Ecole Polytechnique), Palaiseau, France.
[3]Institut FRESNEL (CNRS - Université d'Aix-Marseille), Marseille, France.

*Email: Anne-Sophie.Bonnet-Bendhia@ensta-paristech.fr

Abstract

We consider a Transverse Magnetic time-harmonic scattering problem. The scatterer is a metallic object whose cross-section has corners and whose permittivity is a function of the frequency, typically given by Drude's law. When the dissipation effects in the metal are neglected, it has been proved in [2] that there is a range of frequencies where some energy is trapped by the corners, due to the so-called plasmonic black-hole waves. The purpose of this work is to show that a similar phenomenon can be observed when considering a realistic dissipative metal, like silver.

Keywords: electromagnetic scattering, Drude's model, singularities, black-hole waves, energy balance

1 The scattering problem

For simplicity, let us suppose that the cross-section Ω of the metallic scatterer has the shape of a droplet (see figure 1), with a single corner, located at the origin. The relative dielectric permittivity in the metal obeys the following law, known as the Drude's model

$$\varepsilon_\gamma(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}, \qquad (1)$$

where $\omega > 0$ is the pulsation, $\gamma \geq 0$ (for a harmonic regime in $e^{-i\omega t}$) characterizes the dissipative effects, and $\omega_p > 0$ is the plasma frequency (for silver $\gamma = 0.113\,10^{15}$ Hz and $\omega_p = 13.3\,10^{15}$ Hz [1]). We are interested in a frequency range $\omega < \omega_p$ below the plasma frequency, where the real part of $\varepsilon_\gamma(\omega)$ is negative while its imaginary part is positive (see figure 1). We consider the following scattering problem: find $u_\gamma = u^{\mathrm{inc}} + u_\gamma^{\mathrm{sca}}$ such that $\operatorname{div}\left(\varepsilon_\gamma^{-1}\nabla u_\gamma\right) + \omega^2 c^{-2} u_\gamma = 0$ in $\mathbb{R}^2$ and

$$\lim_{\xi\to+\infty} \int_{|\mathbf{x}|=\xi} \left|\frac{\partial u_\gamma^{\mathrm{sca}}}{\partial r} - i\omega c^{-1} u_\gamma^{\mathrm{sca}}\right|^2 d\sigma = 0,$$

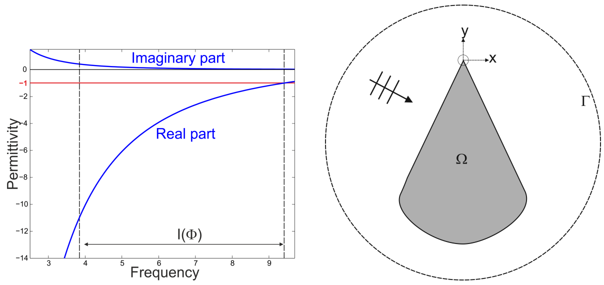

Figure 1: Permittivity of silver and geometry

where u_γ represents the transverse component of the magnetic field, u^{inc} is a plane wave, c denotes the light speed and ε_γ is a function defined by $\varepsilon_\gamma = 1$ in $\mathbb{R}^2 \setminus \Omega$ and $\varepsilon_\gamma = \varepsilon_\gamma(\omega)$ in Ω.

For $\gamma > 0$, thanks to the imaginary part of $\varepsilon_\gamma(\omega)$, one can prove with standard arguments that this problem is well-posed in $H^1_{\mathrm{loc}}(\mathbb{R}^2)$, and if Γ is a circle enclosing the droplet, we have:

$$-\Im m \left(\int_\Gamma \frac{\partial u_\gamma}{\partial r}\overline{u_\gamma}\right) = \Im m \left(\frac{-1}{\varepsilon_\gamma(\omega)}\right) \int_\Omega |\nabla u_\gamma|^2.$$

This quantity, denoted in the sequel by $J_\gamma(\omega)$, is strictly positive and corresponds to the energy dissipated during one time period in the metallic inclusion.

2 The non-dissipative case $\gamma = 0$

If γ is small compared to ω_p, there is a range of frequencies between γ and ω_p where it may be relevant to neglect the dissipation in the metal by taking $\gamma = 0$ (see figure 1). Then the permittivity ε_0 is a real-valued function, negative in the metal and positive elsewhere. The well-posedness of such a sign-changing transmission problem has been extensively studied and the results for the scattering problem depend on the value of ω (see [2] for the details). If we denote by Φ the angle at the corner, we define the frequency interval $I(\Phi)$ by the following property:

$$\omega \in I(\Phi) \Leftrightarrow \varepsilon_0(\omega) \in \left]-\frac{2\pi - \Phi}{\Phi}, -1\right[.$$

If $\omega \notin \overline{I(\Phi)}$, the scattering problem is well-posed in $H^1_{\text{loc}}(\mathbb{R}^2)$ and there is no energy dissipation ($J_0(\omega) = 0$). On the contrary, if $\omega \in I(\Phi)$, the scattering problem is not well-posed in $H^1_{\text{loc}}(\mathbb{R}^2)$. However the well-posedness can be recovered in a different functional framework [3]. The solution u_0 (which is the limit of u_γ when $\gamma \to 0$) may be very singular at the corner, it behaves like $a\exp(i\kappa \log r)$ (in polar coordinates) with $\kappa \in \mathbb{R}$, where the constant $a \in \mathbb{C}$ depends on the incident wave. When $a \neq 0$, this so-called black-hole wave carries energy towards the corner, which results in a dissipation of energy ($J_0(\omega) > 0$), even if the dissipation in the material has been neglected.

3 The slightly dissipative case

From a physical point of view, the relevance of this strange phenomenon (dissipation of energy in a non dissipative material) is discussed in the literature [4]. Indeed one could suspect that it is due to idealized non-realistic hypotheses, like the perfect corner and the non dissipative material ($\gamma = 0$). Our objective here is to show that the phenomenon of leakage at the corner is still present in a realistic dissipative material.

We have computed $J_\gamma(\omega)$ as a function of ω for the case of silver for two different incidences and for two inclusions, a droplet-shaped one as described above, with an angle $\Phi = \pi/6$, while the second one has the shape of a disk. For the comparison, the two shapes have the same perimeter as losses are due to the plasmonic surface wave propagating at the surface of the metal (the wave does not propagate inside the metal because the real part of ε and μ have opposite signs). Obviously, for the disk, the two incidences give the same result represented by the black dashed curve. For the droplet, the first incidence in red is such that the black-hole wave is excited while it is not for the second incidence in blue.

As expected, the energy losses for the droplet are much larger than for the disk in the interval $I(\Phi)$ when the black-hole wave is excited.

Let us mention that a refined mesh near the interface would be necessary for the frequencies at the right end of $I(\Phi)$, which correspond to the almost ill-posed case $\Re e(\varepsilon_\gamma(\omega)) = -1$.

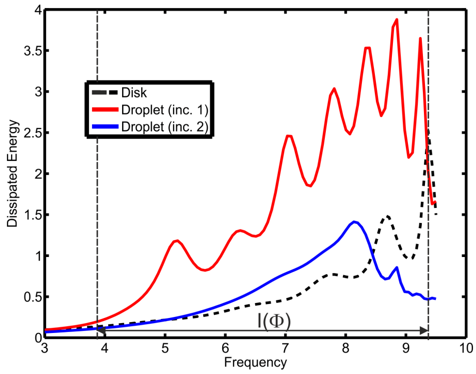

Figure 2: Energy dissipation for a droplet and a disk inclusions, for two directions of incidence

The computations are done with the code Xlife++ [5].

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